Indian Statistical Institute
B.Math I Year
First Semester, 2006-2007
Mid Semester Examination
Probability Theory I

Time: 2 1/2 hrs | Date:18-09-06 | Max. Marks : 80

Note: The paper carries 83 marks. Any score above 80 will be treated as 80.

1. Let $2n \leq r$. Two boxes each have r balls labelled $1, 2, \ldots, r$. A random sample of size n is drawn without replacement from each box. Find the probability that the samples contain exactly k balls having the same numbers in common. [15]

2. Suppose n distinct balls are distributed at random into 3 distinct boxes. Let X = number of balls in Box 1, and Y = number of balls in Box 2.

   (i) Find the discrete density function of (X, Y).

   (ii) Find the marginal density functions.

   (iii) Are X and Y independent? [10+10+3]

3. Show that the distribution function of a real valued discrete random variable is right continuous. [15]

4. Let X and Y be independent integer valued random variables. Show that the discrete random variables X^2 and Y^2 are independent. [15]

5. Let X be a nonnegative integer valued random variable. Let $a > 0$ be a constant. Let $p_n = P(X = n), n = 0, 1, 2, \ldots$. Suppose $\{p_n\}$ satisfy $p_n = \frac{a}{n} p_{n-1}, \quad n = 1, 2, \ldots$

   (i) Find $p_n, \quad n = 0, 1, 2, \ldots$.

   (ii) Find $E(X)$. [10+5]